Flow dimensions corresponding to stochastic models of heterogeneous transmissivity

Douglas D. Walker,[1] Pablo A. Cello,[2] Albert J. Valocchi,[2] and Bruce Loftis[3]

Received 6 January 2006; revised 22 February 2006; accepted 1 March 2006; published 15 April 2006.

[1] Traditional approaches to characterization and modeling of highly heterogeneous aquifers face many technical challenges. One nontraditional approach begins with the Generalized Radial Flow interpretation of hydraulic tests, which infers an additional parameter, the flow dimension, to describe flow geometry. The flow dimension is examined for three stochastic models of heterogeneous transmissivity, T(x), via Monte Carlo analysis of numerical models. For lognormal $T(x)$ of low variance, the ensemble average of the apparent flow dimensions is two regardless of the test duration, and if the test duration is sufficiently long, the apparent flow dimension converges to two even for individual tests. The variability of the apparent flow dimension depends on the variance and integral scale of ln $T(x)$, suggesting that these parameters might be estimated from a set of aquifer tests. Results suggest that the flow dimension may be useful for selecting models of heterogeneity and their parameters.
Citation: Walker, D. D., P. A. Cello, A. J. Valocchi, and B. Loftis (2006), Flow dimensions corresponding to stochastic models of heterogeneous transmissivity, *Geophys. Res. Lett.*, *33*, L07407, doi:10.1029/2006GL025695.

1. Introduction

[2] Fluid flow and contaminant transport in fractured rocks are difficult to characterize and model because their heterogeneity concentrates flow into erratic channels that are poorly suited to traditional analysis techniques [*National Research Council*, 1996]. For such formations, alternative approaches include the Generalized Radial Flow (GRF) approach to the interpretation of aquifer tests [*Barker*, 1988]. The GRF approach interprets an additional parameter from an aquifer test called the flow dimension, denoted n, to describe how the cross-sectional area of flow changes with radial distance from the pumped well. Researchers have observed that heterogeneities can restrict flow such that the flow dimensions are other than the spatial (Euclidean) dimension of the aquifer [*Acuna and Yortsos*, 1995], but have provided little guidance on how the flow dimension might be used.

[3] This study examines the behavior of the flow dimension for stochastic models of aquifer heterogeneity, with the objective of identifying models that produce the noninteger flow dimensions that have been observed in the field. The study consists of a Monte Carlo analysis of a numerical model of an aquifer test, using several widely used stochastic models to simulate heterogeneous fields of transmissivity.

2. Background

[4] The starting point for this study is the log-log diagnostic plot of *Bourdet et al.* [1983] used for the analysis of constant-rate aquifer tests. Figure 1 presents a log-log diagnostic plot of the drawdown, s, and drawdown derivative $s' = (ds/d \ln t)$ versus time for an aquifer test in a fractured dolomite aquifer. This aquifer is laterally extensive, thin, and nonleaky, such that the traditional interpretation would fit the two-dimensional Theis model [*Theis*, 1935] to infer the transmissivity and storage coefficient. However, the Theis model fits these test data poorly, translating into unreliable parameter estimates and increased uncertainties in modeling. For the GRF interpretation, the late-time slope of the log-log plot of the derivative, v, is related to the flow dimension by $n = 2 - 2 \cdot v$ [*Barker*, 1988]. Thus for Figure 1, the derivative has a slope of $v \approx 0.15$, so that $n \approx 1.7$, even though the aquifer arguably is two-dimensional. Excluding boundary effects, flow dimensions other than the Euclidean dimension typically are attributed to unspecified forms of heterogeneity. This study examines several stochastic models of heterogeneity to see if they produce the flow dimensions observed in the field.

[5] Although there have been recent advances in identifying parameters of stochastic models of heterogeneous aquifers using aquifer tests [*Copty and Findikakis*, 2004; *Neuman et al.*, 2004], there are few studies of using the flow dimension to select heterogeneity models. *Barker* [1988] conjectured that non-integer flow dimensions were caused by a fracture network acting as a fractal object. *Polek* [1990] verified Barker's conjecture numerically, and *Acuna and Yortsos* [1995] confirmed that the GRF model was a special case of radial diffusion on a fractal lattice. *Doe* [1991] noted that the interpreted flow dimension might be the consequence of heterogeneity, variations in the flow geometry, or combinations of both. Several studies have suggested that flow and transport models should reproduce the flow dimensions inferred from aquifer tests but do not suggest methods for doing so [*Riemann et al.*, 2002]. *Walker and Roberts* [2003] found that a stationary transmissivity of modest variability eventually shows $n = 2$, but the flow dimension of a nonstationary field depended on the form of nonstationarity.

3. Approach

[6] This study uses an approach similar to *Meier et al.* [1998]: (1) create a transmissivity field $T(x)$ using one of

[1]Illinois State Water Survey, Champaign, Illinois, USA.
[2]Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois, USA.
[3]Rosen Center for Advanced Computing, Purdue University, West Lafayette, Indiana, USA.

0094-8276/06/2006GL025695$05.00

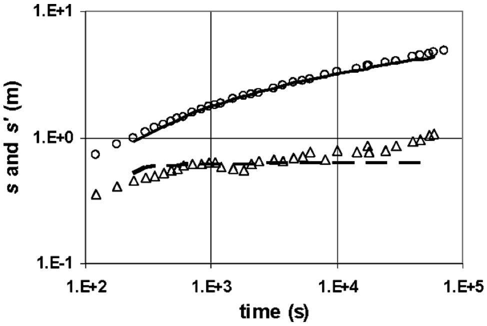

Figure 1. An aquifer test in a fractured dolomite aquifer: Hopkins Park Well #2, Kankakee Co., Illinois, USA (ISWS archives). Circles denote the drawdown, *s*, vs. time; triangles denote the derivative, s'. The Theis model for radial flow is the solid line, and its derivative is the dashed line.

several stochastic models; (2) simulate a constant-rate aquifer test in the field using a numerical model of groundwater flow; and (3) estimate the flow dimension from the resulting drawdowns. The sequence is repeated Monte-Carlo fashion to infer the behavior of the flow dimension for each stochastic model. The aquifer test is simulated with MODFLOW-2000 [*Harbaugh et al.*, 2000], the USGS finite-difference model for transient groundwater flow. The model grid spacing is dense (3001 × 3001 nodes, 1m spacing) to represent the stochastic models accurately and extensive to simulate aquifer tests without boundary effects. Flow dimensions are reported for the centrally located pumping well using finite differences in time to determine the slopes of the drawdown and its derivative. Constant head boundaries are placed around the model exterior so that contact of the aquifer test with the model limits is revealed by a sharp increase in the flow dimension in the final few time steps of the simulation. Checks on simulation accuracy (e.g., flow balance errors, reproduction of the statistical moments and effective transmissivities) and quality assurance checks are presented by *Walker et al.* [2006].

[7] This study takes advantage of the thorough characterization studies of the Culebra Dolomite, a fractured dolomite aquifer found at the Waste Isolation Pilot Plant near Carlsbad, NM [*Beauheim and Ruskauff*, 1998]. The productive interval is the lower 4.4m of the Culebra, whose aquifer tests at the H-11 well cluster suggest an interpreted transmissivity of $T = 4.7 \times 10^{-5}$ m^2/s and a storage coefficient of $S = 4.7 \times 10^{-5}$. *Meier et al.* [1998] found that the interpreted *T* of a well test is a valid approximation for T_g, the geometric mean *T* of the field. For this study, the storage coefficient is assumed to be homogeneous. The pumping rate for the simulated aquifer test is 0.22 L/s.

4. Results

[8] This study considers three stochastic models: the lognormal case, i.e., ln $T(x)$ as a spatially correlated (multivariate) Gaussian field (mvG); fractional Brownian motion (fBm), which is a variant of the mvG model; and a site percolation network with a percolation probability near the critical threshold. *Neuman* [1995] noted that an fBm field requires at least one conditioning value to define its mean and this conditioning value logically is located at the pumped well. To be consistent, all three stochastic models use a conditioning datum at the pumped well of $T = T_g$. For some systems, the slope of the derivative of the log-log diagnostic plot varies over time, such that the flow dimension varies as the aquifer test evolves, thus we plot the *apparent* flow dimension, $n^* = 2 - 2 \cdot \nu^*$:

$$\nu^*(t) = \frac{d}{d(\log t)}\left[\log\left({}^{dh}/_{d\ln t}\right)\right] \quad (1)$$

Monte Carlo estimates of the arithmetic averages, medians, and variances of the apparent flow dimension are approximately stable for 100 realizations for mvG and fBm models, and 200 realizations for the percolation model.

4.1. Multivariate Gaussian (mvG)

[9] The mvG fields are created using sequential simulation [*Deutsch and Journel*, 1998], with an exponential semivariogram. Figure 2 shows that the arithmetic average of the apparent flow dimension converges rapidly to $n^* = 2$ for 100 realizations using a variance of $\sigma^2_{\ln T} = 0.25$ and an integral scale $I = 7$m. The variability between realizations decreases with time, suggesting that even individual aquifer tests in an mvG field will tend to show $n = 2$ if the radius of investigation is much greater than the integral scale. The decreasing variability of n^* occurs well in advance of the contact with the domain boundary (indicated in Figure 2 by the abrupt increase in the mean of n^* during the final few time steps), suggesting that the decreasing variability of n^* is not an artifact of the finite domain. Decreasing the integral scale to 3.5m without changing the variance accelerates the convergence of the apparent flow dimension. Increasing the variance to 1.0 increases the variance of the apparent flow dimension (Figure 3). These dependencies suggest that it may be possible to identify the variance and integral scale of a mvG field from a set of aquifer tests, similar to *Copty and Findikakis* [2004].

4.2. Fractional Brownian Motion (fBm)

[10] The fractional Brownian motion (fBm) model is simply the mvG model with a power model for the semivariogram:

$$\gamma(h) = C_1 h^{2H} \quad (2)$$

where $\gamma(h)$ is the semivariogram of ln $T(x)$, *h* is the absolute separation (lag) distance between two points x_1 and x_2, *H* is

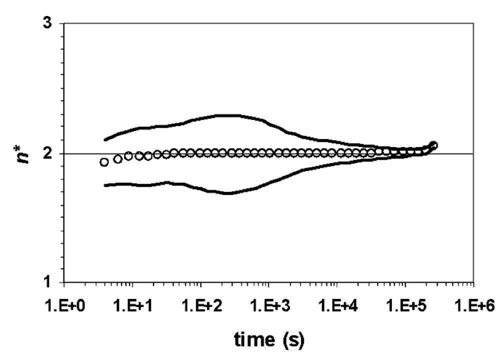

Figure 2. Average (circle) and 95% normal CI for the population for 100 realizations of the flow dimension: mvG with $\sigma^2_{\ln T} = 0.25$ and $I = 7$m.

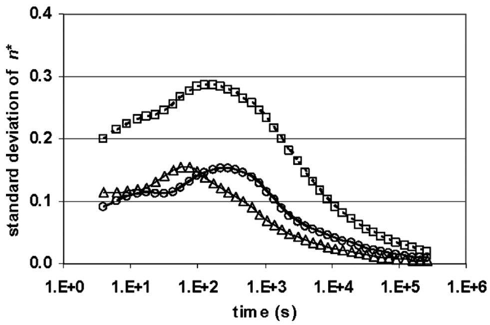

Figure 3. Standard deviation of apparent flow dimensions for a constant-rate aquifer test for the mvG model. Circles: 100 realizations with $\sigma^2_{\ln T} = 0.25$ and $I = 7$m; triangles: 100 realizations with $\sigma^2_{\ln T} = 0.25$ and $I = 3.5$m; squares: 1000 realizations with $\sigma^2_{\ln T} = 1.0$ and $I = 7$m.

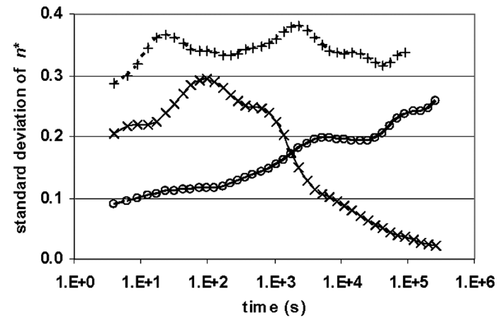

Figure 5. Standard deviation of apparent flow dimensions for a constant-rate aquifer test: × = 100 realizations of *mvG with* $\sigma^2_{\ln T} = 1.0$ and $I = 7$m; o = 100 realizations of fBm with $H = 0.25$; and + = 200 realizations of a site percolation network with $p = 0.61$.

the Hurst coefficient, and C_1 is a scaling constant; *Neuman* [1994] has argued that $H = 0.25$ and $C_1 = 0.027$ in general, and these values are used in this study.

[11] The arithmetic average of the apparent flow dimension for the fBm model is $n^* = 2$ (Figure 4), and the variability of the apparent flow dimension increases with time (Figure 5). The increasing variability of the apparent flow dimension is attributable to the increasing variance of ln $T(x)$ with scale (Equation 2), and is enhanced by the conditioning value which restricts the variability of ln $T(x)$ near the well. The increasing variability of the apparent flow dimension contrasts with that of the mvG model, which suggests that the variability of the flow dimension of a set of aquifer tests might be used to distinguish between the fBm and mvG models.

4.3. Percolation Network

[12] A percolation network idealizes a porous medium as a lattice with a probability of p that a location in a lattice conducts flow. As p increases from zero to the critical probability, p_c, the interconnected sites on the lattice finally grow large enough to span the domain and flow can percolate across the system. Percolation clusters with $p \approx p_c$ have been shown to have fractal geometries, and they are the subject of much study [*Berkowitz and Balberg*, 1993].

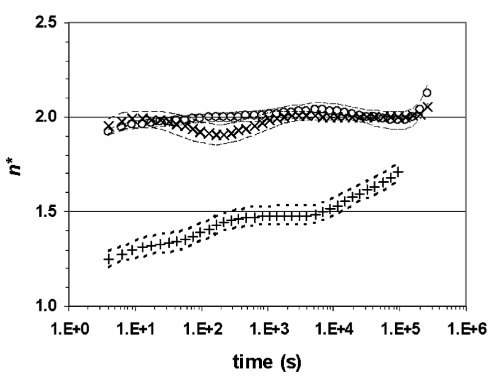

Figure 4. Apparent flow dimensions for a constant-rate aquifer test for three stochastic models. The average and 95% normal confidence interval for the mean for: × = 100 realizations of mvG with $\sigma^2_{\ln T} = 1.0$ and $I = 7$m; o = 100 realizations of fBm with $H = 0.25$; and + = 200 realizations of a site percolation network with $p = 0.61$.

[13] In percolation network terms, the node-centered grid of MODFLOW-2000 is a quadratic, site-percolation network with a critical value of $p_c \approx 0.593$ [*Stauffer and Aharony*, 1994]. For this study, a random proportion $p = 0.61$ of the grid nodes are selected to conduct flow, and since p is slightly greater than p_c, the resulting network should have fractal properties and tend to span the domain. The percolating nodes are assigned $T_p = T_g$, and the remaining (nonpercolating) nodes are coded as no-flow cells. Percolation networks have a correlation length, ξ_p, defined for a two-dimensional lattice as [*Stauffer and Aharony*, 1994]:

$$\xi_p = \Delta x \cdot |p - p_c|^{-4/3} \tag{3}$$

At length scales between Δx and ξ_p, the percolation network has fractal properties; at scales greater than ξ_p, the percolation network will behave as a two-dimensional field [*Polek*, 1990]. For this grid, $\xi_p = 230$m, so that the apparent flow dimensions of this model might transition to $n^* = 2$ when the radius of influence of the test becomes larger than ξ_p. The node representing the pumping well is a percolating node in all realizations, but it is not a 'conditioning value', since this model does not include spatial correlation. Some realizations are of wells pumped in small percolation clusters that are overwhelmed by contact with limits of the cluster. For the purposes of analyzing the apparent flow dimension of the heterogeneity, these finite clusters are trimmed from the set of realizations. Including such realizations result in only minor changes in the average of the apparent flow dimensions.

[14] The average of the apparent flow dimension of the percolation model temporarily stabilizes to 1.5, then steadily increases (Figure 4) while its variance is roughly constant (Figure 5). The late-time increase in n^* is attributed to the radius of influence growing larger than ξ_p and the network becoming progressively more two-dimensional.

5. Summary and Conclusion

[15] Figure 4 compares the flow dimension results for the stochastic models, with the arithmetic average of the appar-

ent flow dimension at the pumped well plotted versus time since the start of the test. The averages of the apparent flow dimensions of the mvG and fBm cases appear to stabilize to approximately $n^* = 2$, while that of the percolation network appears to stabilize to $n^* = 1.5$. The variability of the apparent flow dimension decreases over time for the mvG model, is roughly constant for the percolation network, and steadily increases for the fBm model (Figure 5). These differences suggest that it may be possible to use the variability and average of the flow dimension of a set of aquifer tests to differentiate between alternative models of heterogeneity and estimate their parameters. While the confidence intervals for the low variance mvG and fBm models indicate that individual aquifer tests might have $n* < 2$ for short intervals, these models on average do not produce flow dimensions less than the Euclidean dimension of the aquifer. Preliminary analyses of fractured dolomites in Illinois (e.g., Figure 1) and in New Mexico show flow dimensions between 1.4 and 2.0, suggesting that the percolation network model is the most appropriate for representing fractured dolomite aquifers.

[16] In a practical context, the individual realizations for a particular stochastic model are analogous to non-overlapping aquifer tests scattered throughout a large aquifer, and the averages taken over the set of realizations (e.g., Figure 3) are analogous to the inferred properties of that large aquifer. If the average flow dimension of a set of aquifer tests is $n^* = 1.5$, then the mvG model of low variance is a poor choice for representing that aquifer. The results also show that even *individual* aquifer tests of sufficient duration converge to apparent flow dimensions of $n^* = 2$ for an mvG aquifer, so that an individual aquifer test of long duration with, for example, $n^* = 1.5$ also indicates that the mvG model is a poor choice for representing that aquifer.

[17] These results suggest that the flow dimension may be a useful diagnostic for selecting models of heterogeneity. Additional research is advocated to infer the general behavior of the flow dimension at various field sites, to examine other stochastic models and a broader range of parameters, and to explore the relationship between transport behavior and the flow dimension.

[18] **Acknowledgments.** This material is based upon work supported by Sandia National Laboratories under SNL PO 24699, the Illinois Water Resources Center under Award Number 01HQGR0112; the National Center for Supercomputing Applications under the Faculty Fellows Program; and the Illinois State Water Survey. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the sponsors. The authors acknowledge the assistance of Steffan Mehl (U.S. Geological Survey); Susan Altman, Rick Beauheim, Glenn Hammond, Sean McKenna, and Randy Roberts (Sandia National Laboratories) for discussion and debate regarding this research. The review comments of Al Wehrmann and Mark Anliker (ISWS), and Xavier Sánchez-Vila (Universidad Politécnica de Catalunya) led to substantial improvements and are gratefully acknowledged.

References

Acuna, J. A., and Y. C. Yortsos (1995), Application of fractal geometry to the study of networks of fractures and their pressure transient, *Water Resour. Res.*, *31*, 527–540.

Barker, J. A. (1988), A generalized radial-flow model for pumping tests in fractured rock, *Water Resour. Res.*, *24*, 1796–1804.

Beauheim, R. L., and G. J. Ruskauff (1998), Analysis of Hydraulic Tests of the Culebra and Magenta dolomites and Dewey Lake redbeds conducted at the Waste Isolation Pilot Plant site, *SAND98–0049*, Sandia Natl. Lab., Albuquerque, N. M.

Berkowitz, B., and I. Balberg (1993), Percolation theory and its application to groundwater hydrology, *Water Resour. Res.*, *29*, 775–794.

Bourdet, D., T. M. Whittle, A. A. Douglas, and Y. M. Pirard (1983), A new set of type curves simplifies well test analysis, *World Oil, May*, *196*, 95–106.

Copty, N. K., and A. Findikakis (2004), Bayesian identification of the local transmissivity using time-drawdown data from pumping tests, *Water Resour. Res.*, *40*(12), W12408, doi:10.1029/2004WR003354.

Deutsch, C. V., and A. G. Journel (1998), *GSLIB: Geostatistical Software Library and User's Guide*, 2nd ed., 369 pp., Oxford Univ. Press, New York.

Doe, T. W. (1991), Fractional dimension analysis of constant-pressure well tests, paper presented at 1991 SPE Annual Technical Conference and Exhibition, 1991, Soc. Pet. Eng., Dallas, Tex.

Harbaugh, A. W., E. R. Banta, M. C. Hill, and M. G. McDonald (2000), Modflow-2000, the U.S. Geological Survey modular ground-water model: User guide to modularization concepts and the ground-water flow process, *U.S. Geol. Surv. Open File Rep.*, *20000-92*, 121 pp.

Meier, P. M., J. Carrera, and X. Sánchez-Vila (1998), An evaluation of Jacob's method for the interpretation of pumping tests in heterogeneous formations, *Water Resour. Res.*, *34*, 1011–1025.

National Research Council (1996), *Rock Fractures and Fluid Flow: Contemporary Understanding and Applications*, 551 pp., Washington, D. C.

Neuman, S. P. (1994), Generalized scaling of permeabilities: Validation and effect of support scale, *Geophys. Res. Lett.*, *21*, 349–352.

Neuman, S. P. (1995), On advective transport in fractal permeability and velocity fields, *Water Resour. Res.*, *31*, 1455–1460.

Neuman, S., A. Guadagnini, and M. Riva (2004), Type-curve estimation of statistical heterogeneity, *Water Resour. Res.*, *40*, W04201, doi:10.1029/2003WR002405.

Polek, J. M. (1990), Studies of the hydraulic behavior of hierarchically fractured rock geometries, M.S. thesis, 72 pp., Univ. of Calif, Berkeley.

Riemann, K., G. van Tonder, and P. Dzanga (2002), Interpretation of single-well tracer tests using fractional flow dimensions. part 2: A case study, *Hydrogeol. J.*, *10*, 357–367.

Stauffer, D., and A. Aharony (1994), *Introduction to Percolation Theory*, 2nd ed., 181 pp., Taylor and Francis, Philadelphia, Pa.

Theis, C. (1935), The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage, *Eos Trans. AGU*, *16*, 519–524.

Walker, D. D., and R. M. Roberts (2003), Flow dimensions corresponding to hydrogeologic conditions, *Water Resour. Res.*, *39*(12), 1349, doi:10.1029/2002WR001511.

Walker, D. D., P. A. Cello, A. J. Valocchi, and B. Loftis (2006), High-throughput computing for the analysis of tracer tests in fractured aquifers, Contract Report 2006-04, Ill. State Water Surv., Champaign, in press.

P. A. Cello and A. J. Valocchi, Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA.

B. Loftis, Rosen Center for Advanced Computing, Purdue University, West Lafayette, IN 47907, USA.

D. D. Walker, Illinois State Water Survey, Champaign, IL 61820–7497, USA. (ddwalker@uiuc.edu)