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Analytical solutions for transport in porous media with Gaussian source terms

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Abstract. Analytical solutions for the advective-dispersion equation for solute transport in porous media commonly assume a uniform distribution of mass within the source term. This paper derives an analytical solution for transport in porous media for a source term whose mass is distributed as a bivariate Gaussian spatial function. The solution is an extension of existing analytical solutions using a Green's function approach to separate out one-dimensional terms in a manner similar to previous authors. This approach illustrates the relationship of the bivariate Gaussian source term solution to other Green's function solutions and thus leads to a set of solutions for advective-dispersive transport with various source term and domain geometries. Comparison of point, bivariate Gaussian, and uniform source term solutions finds the greatest differences near the source, with discrepancies decreasing with travel distance.

1. Introduction

Analytical solutions have been widely used in engineering to model heat and contaminant transport. *Carslaw and Jaeger* [1959], for example, present solutions for heat transport which can be adapted to solute transported by advection and Fickian dispersion in a uniform flow field. The governing equation for such advective-dispersive transport in three dimensions can be written as (similar to *Bear* [1972])

$$\frac{\partial c}{\partial t} = D^*_{xx}\frac{\partial^2 c}{\partial x^2} + D^*_{yy}\frac{\partial^2 c}{\partial y^2} + D^*_{zz}\frac{\partial^2 c}{\partial z^2} - v^*\frac{\partial c}{\partial x} - \lambda c, \quad (1)$$

where c is the solute concentration in water; $D^*_{xx} = \alpha_x v/R$, $D^*_{yy} = \alpha_y v/R$, and $D^*_{zz} = \alpha_z v/R$ are the retarded hydrodynamic dispersion in the x, y, and z directions, respectively, with dispersivities α_x, α_y, and α_z; v^* is the retarded, uniform, steady seepage velocity in the x direction (i.e., $v^* = -K(\partial h/\partial x)/(\phi R)$); $\lambda = -\ln 2/t_{1/2}$ is the decay constant with $t_{1/2}$ equal to contaminant half-life; $R = (1 + \rho_b k_d/\phi)$ is the retardation coefficient; ϕ is the porosity; ρ_b is the bulk density; and k_d is linear adsorption coefficient. The boundary conditions and initial conditions are

$$c(x, y, z, 0) = 0, \quad (2)$$

$$c(\pm\infty, y, z, t) = 0, \quad (3)$$

$$c(x, \pm\infty, z, t) = 0, \quad (4)$$

$$-D^*_{zz}\left.\frac{\partial c}{\partial z}\right|_{z=0} = 0, \quad (5a)$$

$$c(x, y, \infty, t) = 0, \quad (5b)$$

$$c(x, y, z, 0) = \frac{1}{\phi R} m(\xi, \eta, \zeta, 0) \qquad (x, y, z) \in (\xi, \eta, \zeta). \quad (6)$$

Equation (6) represents a mass of contaminant that appears instantaneously in the source region $(x, y, z) \in (\xi, \eta, \zeta)$, where $m(\ )$ is the bulk concentration of contaminant (i.e., the mass in all phases per unit bulk volume). The boundary conditions given by (3) through (6) may be also be written as combinations of bounded domains, each with a different solution.

Various mathematical strategies have been employed to solve (1). One strategy is to consider (1) as a whole and solve it for specific boundary conditions using integral transforms. This approach has led to a number of analytical solutions for (1) through (6), each for a particular combination of source term and boundary conditions [e.g., *Gureghian*, 1987; *Wexler*, 1989; *Leij et al.*, 1991]. However, solving (1) as a whole obscures the relationships between solutions for similar boundary conditions; intuitively, one would expect that the solution of (1) for infinite x, y, and z could be developed from a similar solution for bounded z. An alternative solution strategy separates (1) through (6) into x, y, and z terms and then solves the resulting one-dimensional equations via Green's functions; the three-dimensional solution is simply the product of these one-dimensional solutions [*Carslaw and Jaeger*, 1959]. The strength of this approach is that the one-dimensional solutions may be recombined to create a large set of two- and three-dimensional solutions for a variety of source term and domain geometries [e.g., *Yeh and Tsai*, 1976; *Codell et al.*, 1982]. Most of the preceeding analytical solutions assume a uniform distribution of contaminant mass within the source. The exceptions are the analytical solutions of *Cleary and Ungs* [1978], *Gureghian* [1987], and *Serrano* [1996], which were developed for source terms with Gaussian mass distributions.

This paper presents Green's function solutions for advective-dispersive transport in porous media from a source term on the media's surface (i.e., on the water table of an aquifer)

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with a bivariate Gaussian distribution of mass in space. These solutions can be seen as extensions of *Cleary and Ungs'* [1978] univariate Gaussian source term solution to bivariate Gaussian source terms, generalized similar to *Yeh and Tsai* [1976] using Green's functions. We compare a particular case of a bivariate Gaussian source solution to point and uniform source solutions.

2. Solute Transport and Green's Functions

The spatial distribution of mass in the source region (equation (6)) may be any arbitrary function, but for the purposes of this paper it is treated as a separable function of the spatial variables. That is, the total mass M appearing instantaneously in the source region can be written

$$M = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{0}^{b} m(\xi, \eta, \zeta, 0)\, d\xi\, d\eta\, d\zeta$$

$$= \int_{-\infty}^{+\infty} m(\xi, 0)\, d\xi \int_{-\infty}^{+\infty} m(\eta, 0)\, d\eta \int_{0}^{+\infty} m(\zeta, 0)\, d\zeta.$$

For the boundary and initial conditions given by (2) through (6), various texts [e.g., *Haberman*, 1983] present a Green's function solution for $c_i(x, y, z, t)$, the concentration due to an instantaneous source, as

$$c_i(x, y, z, t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{0}^{\infty} \frac{1}{\phi R} m(\xi, 0) m(\eta, 0) m(\zeta, 0)$$
$$\cdot G(x, t|\xi, 0) G(y, t|\eta, 0) G(z, t|\zeta, 0)\, d\xi\, d\eta\, d\zeta e^{-\lambda t}. \tag{7}$$

The terms $G(x, t|\xi, 0)$, $G(y, t|\eta, 0)$, and $G(z, t|\zeta, 0)$ are the Green's functions in the x, y, and z directions, respectively. The Green's functions describe the system response to the instantaneous appearance of mass in the source region. The Green's functions corresponding to (1) subject to (2) through (6) can be found via Fourier analysis to be

$$G(x, t|\xi, 0) = \frac{1}{\sqrt{4\pi D_x^* t}} \exp\left[-\frac{(x - v^*t - \xi)^2}{4D_x^* t}\right], \tag{8a}$$

$$G(y, t|\eta, 0) = \frac{1}{\sqrt{4\pi D_y^* t}} \exp\left[-\frac{(y - \eta)^2}{4D_y^* t}\right], \tag{8b}$$

$$G(z, t|\zeta, 0) = \frac{2}{\sqrt{4\pi D_z^* t}} \exp\left[-\frac{(z - \zeta)^2}{4D_z^* t}\right]. \tag{8c}$$

In (8c) the factor of 2 in the numerator accounts for the water table, which prohibits upward dispersion of the contaminant. The advantage of the Green's function approach can be seen rearranging (7) as

$$c_i(x, y, z, t) = \frac{1}{\phi R} X_d\, Y_d\, Z_d\, e^{-\lambda t}, \tag{9a}$$

$$X_d = \int_{-\infty}^{+\infty} m(\xi, 0)_d\, G(x, t|\xi, 0)\, d\xi, \tag{9b}$$

$$Y_d = \int_{-\infty}^{+\infty} m(\eta, 0)_d\, G(y, t|\eta, 0)\, d\eta, \tag{9c}$$

$$Z_d = \int_{0}^{+\infty} m(\zeta, 0)_d\, G(z, t|\zeta, 0)\, d\zeta. \tag{9d}$$

Similar to *Yeh* [1981], the subscript d identifies a particular combination of source term and boundary conditions. Examples of solutions for (9b), (9c), and (9d) are given in Table 1 for source terms with point and uniform mass distributions in domains bounded and unbounded in the z dimension. Finally, the solution for a time continuous source term can be found by applying the convolution theorem to the instantaneous source solution of (9a):

$$c(x, y, z, t) = \int_0^t c_i(x, y, z, t - \omega)\, d\omega. \tag{10}$$

That is, the solution for a time continuous source term is the time convolution of the solution for an instantaneous source. Because the mass M may vary from one instant to the next, (10) can easily represent a time-variant source strength. The convolution integral (equation (10)) is usually solved numerically, with some notable exceptions [e.g., *Wilson and Miller*, 1978]. *Yeh and Tsai* [1976] illustrate how such solutions as (9a) and (10) can be extended to bounded domains, time-variant velocities, and nonconservative transport. *Harada et al.* [1980] included the daughter products of radioactive chain decay, and *Aral and Liao* [1996], *Barry and Sposito* [1989], and *Serrano* [1996] present solutions that include time-variant dispersion.

3. Alternative Source Term Representations

Typically, $m(\xi, 0)_d$, $m(\eta, 0)_d$, and $m(\zeta, 0)_d$ are defined as Dirac or Heaviside functions, corresponding to source terms that are either points or uniform distributions of mass over simple geometries (e.g., lines, rectangular areas, or parallelepiped volumes). The use of uniform mass distributions within the source is not a limitation of the Green's functions themselves but rather is a limitation imposed by the tractability of the integrals in (9b), (9c), and (9d). For example, consider a source term with an instantaneous bivariate Gaussian mass distribution in x and y:

$$m(\xi, 0)_5 m(\eta, 0)_5 = m_x m_y \exp\left[-\frac{\xi^2}{2\sigma_x^2} - \frac{\eta^2}{2\sigma_y^2}\right], \tag{11}$$

where $m_x\, m_y$ is the maximum of the distribution (logically, $m_x = m_y$) and the subscript $d = 5$ is chosen for compatability with the notation of *Yeh and Tsai* [1976]. Separating variables and substituting into (9b), the resulting integral can be solved to

$$X_5 = \frac{m_x \sigma_x}{\sqrt{2D_x^* t + \sigma_x^2}} \exp\left[-\frac{1}{2}\frac{(x - v^*t)^2}{(2D_x^* t + \sigma_x^2)}\right]. \tag{12}$$

(Appendix A provides the details of the derivation.) Similarly, for the y component,

$$Y_5 = \frac{m_y \sigma_y}{\sqrt{2D_y^* t + \sigma_y^2}} \exp\left[-\frac{1}{2}\frac{y^2}{(2D_y^* t + \sigma_y^2)}\right]. \tag{13}$$

Table 1. One-Dimensional Green's Function Solution Examples for Selected Boundary Conditions and Source Geometries

Solution	Source
x direction	
$X_1 = \frac{1}{\sqrt{4\pi D_x^* t}} \exp\left[-\frac{(x - v^* t)^2}{4D_x^* t}\right]$	point source [*Codell et al.*, 1982]
$X_4 = \frac{1}{2L}\left[\operatorname{erf}\left(\frac{x - v^* t + L/2}{\sqrt{4D_x^* t}}\right) - \operatorname{erf}\left(\frac{x - v^* t - L/2}{\sqrt{4D_x^* t}}\right)\right]$	line source of length L [*Yeh*, 1981]
y direction	
$Y_1 = \frac{1}{\sqrt{4\pi D_y^* t}} \exp\left[-\frac{y^2}{4D_y^* t}\right]$	point source in aquifer of infinite extent [*Codell et al.*, 1982]
$Y_4 = \frac{1}{2W}\left[\operatorname{erf}\left(\frac{y + W/2}{\sqrt{4D_y^* t}}\right) - \operatorname{erf}\left(\frac{x - W/2}{\sqrt{4D_y^* t}}\right)\right]$	line source of width W in aquifer of infinite width [*Yeh*, 1981]
z direction	
$Z_0 = 1/b$	line source in z direction, length h = aquifer thickness b
$Z_1 = \frac{1}{b}\left\{1 + 2\sum_{m=1}^{\infty} \cos\left(m\pi\frac{z_s}{b}\right) \cos\left(m\pi\frac{z}{b}\right) \exp\left[-\left(\frac{m\pi}{b}\right)^2 D_z^* t\right]\right\}$	point source in z direction in aquifer of thickness b [*Codell et al.*, 1982]
$Z_2 = \frac{1}{h_2 - h_1}\left\{\frac{h_2 - h_1}{b} + \frac{2}{b}\sum_{m=1}^{\infty}\left(\frac{b}{m\pi}\right)\cos\left(\frac{m\pi z}{b}\right)\left[\sin\left(\frac{m\pi h_2}{b}\right) - \sin\left(\frac{m\pi h_1}{b}\right)\right]\exp\left[-\left(\frac{m\pi}{b}\right)^2 D_z^* t\right]\right\}$	line source from $z = h_1$ to $z = h_2$ in aquifer of thickness b [*Yeh and Tsai*, 1976]
$Z_3 = \frac{1}{\sqrt{\pi D_z^* t}} \exp\left[-\frac{z^2}{4D_z^* t}\right]$	point source in aquifer of infinite depth [*Yeh*, 1981]
$Z_4 = \frac{1}{h}\left[\operatorname{erf}\left(\frac{z + h}{\sqrt{4D_z^* t}}\right) - \operatorname{erf}\left(\frac{z - h}{\sqrt{4D_z^* t}}\right)\right]$	line source from $z = 0$ to $z = h$ in aquifer of infinite depth [*Yeh*, 1981]

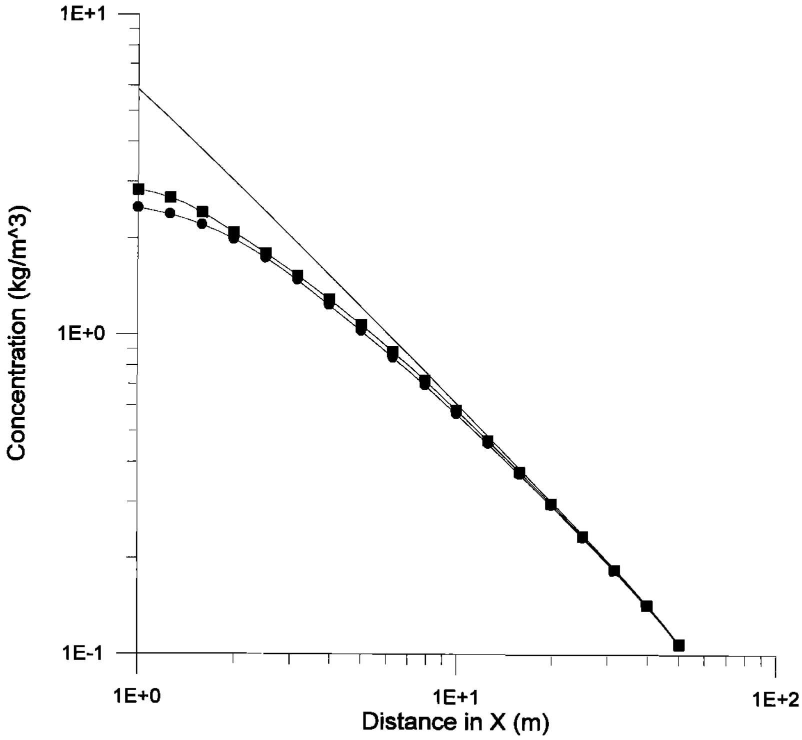

Figure 1. Concentrations along the plume centerline (the line $y = 0$ m and $z = 0.1$ m) at $t = 1000$ days. The plain curve denotes the point source solution, squares denote the uniform plane source solution, and circles denote the bivariate Gaussian source solution.

These solutions may be used with those in Table 1 to construct, for example, the solution for a fully penetrating, instantaneous bivariate Gaussian source (i.e., two-dimensional (2-D) transport):

$$c_i(x, y, z, t) = \frac{1}{\phi R} X_5 Y_5 Z_0 e^{-\lambda t}. \qquad (14)$$

Equation (14) is not, strictly speaking, an original solution since it can be reduced from the 2-D solution for heterogeneous media presented by *Serrano* [1996, equation (32)]. However, the advantage of the Green's function approach is that it is a simple matter to generate solutions for other source term and domain geometries, such as for an infinitely thin, bivariate Gaussian source region on the surface of a infinitely thick aquifer,

$$c_i(x, y, z, t) = \frac{1}{\phi R} X_5 Y_5 Z_3 e^{-\lambda t}, \qquad (15)$$

or for the same source region on an aquifer of finite thickness,

$$c_i(x, y, z, t) = \frac{1}{\phi R} X_5 Y_5 Z_1 e^{-\lambda t}. \qquad (16)$$

Equations (15) and (16) may be further permuted using terms Z_2 and Z_4 from Table 1, creating two additional solutions for bivariate Gaussian sources with finite thickness h in the z direction.

4. Comparison to Other Source Terms

The effects of alternative source term geometries and initial mass distributions can be assessed by comparing the alternative solutions along profiles through the domain. The solutions for three alternative source terms are compared: (1) a continuous point source at $(x, y, z) = (0, 0, 0)$, whose solution is the convolution of

$$c_i(x, y, z, t) = \frac{1}{\phi R} X_1 Y_1 Z_1 e^{-\lambda t}, \qquad (17)$$

(2) a continuous uniform mass distribution over a square source area (3 m by 3 m) in the x-y plane, centered at $(x, y, z) = (0, 0, 0)$, whose solution is the convolution of

$$c_i(x, y, z, t) = \frac{1}{\phi R} X_4 Y_4 Z_1 e^{-\lambda t}, \qquad (18)$$

(3) and a continuous bivariate Gaussian mass distribution over in the x-y plane, centered at $(x, y, z) = (0, 0, 0)$, whose solution is the convolution of (15). Each solution uses an instantaneous mass of $M = 1$ kg, distributed over the source term. In the case of the bivariate Gaussian source we set the maximum of the mass distribution equal to the uniform mass distribution of the plane source, i.e., $M/LW = m_x m_y$, where $L = W = 3$ m. Integrating over the bivariate Gaussian source term shows that $M = m_x m_y \sigma_x \sigma_y 2\pi$, and thus by substitution $\sigma_x = \sigma_y = \sqrt{LW/2\pi} \approx 1.19683$. All three solutions use $\alpha_x = 10.0$ m, $\alpha_y = 1.0$ m, $\alpha_z = 1.0$ m, $v^* = 0.288$ m/d, $t_{1/2} = 1 \times 10^9$ days (i.e., no decay), $R = 1.0$ (i.e., no retardation), and $\phi = 0.3$.

Figure 1 presents the concentrations for each solution at $t = 1000$ days along the plume and centerline (the line $y = 0.0$ m and $z = 0.1$ m). The point source solution is distinctly different from the distributed-mass source solutions, with the differences diminishing with distance. Along this profile the maximum difference between the uniform plane and bivariate Gaussian source solutions is approximately 10–20%. The dif-

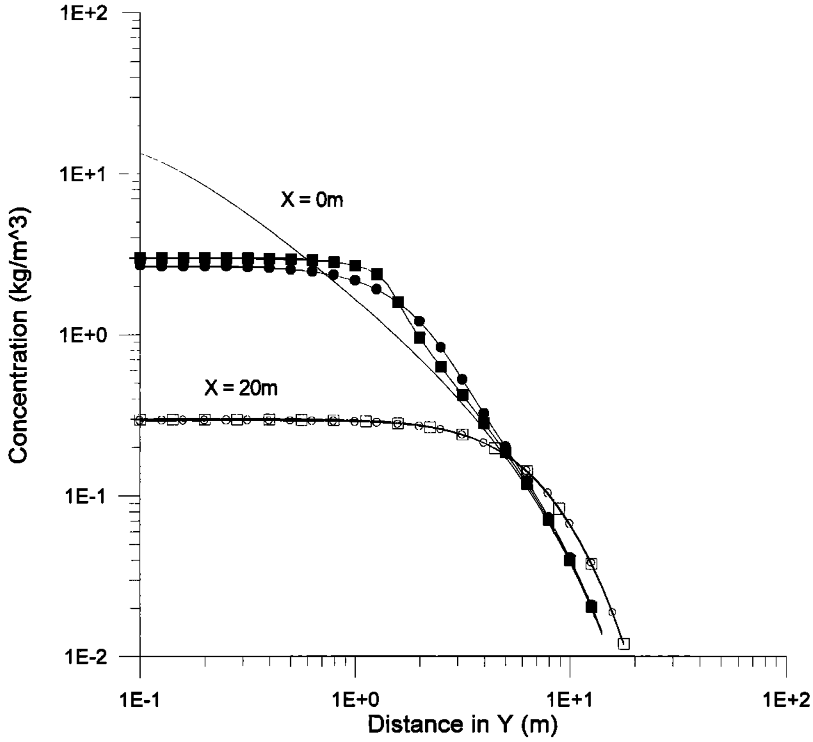

Figure 2. Concentrations at $t = 1000$ days perpendicular to the plume centerline (along the line $x = 0$ m and $z = 0.1$ m and along the line $x = 20$ m and $z = 0.1$ m). At $x = 0$ m the plain line denotes the point source solution, solid squares denote the uniform plane source solution, and solid circles denote the bivariate Gaussian source solution. At $x = 20$ m the plain line denotes the point source solution, open squares denote the uniform plane source solution, and open circles denote the bivariate Gaussian source solution.

ferences diminish moving laterally past the edge of the planar source term (at $x = 1.5$ m), such that all three solutions are virtually indistinguishable at $x = 20$ m. This is attributed to the effects of hydrodynamic dispersion; thus the difference among solutions depends on both the dispersivities and the travel distance.

Figure 2 also presents the concentrations for each solution at $t = 1000$ days but in two cross sections perpendicular to the plume centerline (along the line $x = 0.0$ m and $z = 0.1$ m and along the line $x = 20$ m and $z = 0.1$ m). Again, the point source solution is distinctly different from the solutions of either of the distributed-mass sources, with the differences diminishing with distance from the centerline (in the y direction). Along $x = 0.0$ m the maximum difference of 20–30% between the uniform plane and bivariate Gaussian source solutions appears just beyond the edge of the planar source (i.e., $x = 1.5$–4 m). The second profile of Figure 2 at $x = 20$ m shows that the differences between the solutions diminish rapidly with travel distance (in the x direction).

5. Discussion and Conclusions

Using a Green's function approach, we have shown the relationship between the analytical solutions for the advective-dispersive transport equation for Gaussian and uniform source terms. In doing so, we have shown how Green's function solutions corresponding to Gaussian source terms can be combined with those of, for example, *Yeh and Tsai* [1976] to create a variety of solutions for advective-dispersive transport from nonuniform distributions of mass in the source region. Equations (15) and (16) (and their permutations to sources with finite thickness in z) are believed to be original solutions. However, after more than 150 years of Fourier analysis and Green's function solutions, it seems possible that these solutions have been derived previously. In addition, it might be argued that such solutions could be obtained trivially by inspecting and comparing the solutions of *Yeh and Tsai* [1976], *Cleary and Ungs* [1978], and *Gureghian* [1987]. The solutions presented in this paper, nonetheless, were not found in an extensive literature review. A comparison of solutions for point, uniform planar, and bivariate Gaussian sources reveals that the differences between the solutions is greatest near the source, diminishing with travel distance.

Appendix A: Proof

After separating (11) and substitution into (9b), the integral in x is

$$X_5 = \int_{-\infty}^{+\infty} m_x \exp\left[-\frac{\xi^2}{2\sigma_x^2}\right] \frac{1}{\sqrt{4\pi D_x^* t}} \cdot \exp\left[-\frac{(x - v^* t - \xi)^2}{4D_x^* t}\right] d\xi, \tag{A1}$$

which may be rearranged as

$$X_5 = \frac{m_x}{\sqrt{4\pi D_x^* t}} \int_{-\infty}^{+\infty} \exp\left[-\frac{(x - v^* t - \xi)^2}{4D_x^* t} - \frac{\xi^2}{2\sigma_x^2}\right] d\xi. \tag{A2}$$

Expanding, factoring, and completing the square converts the exponential term of (A2) to

$$\exp\left\{-\frac{1}{4}\frac{\left[\xi+\frac{\sigma_x^2(\nu^*t-x)}{(2D_x^*t+\sigma_x^2)}\right]^2}{\left[\frac{\sigma_x^2D_x^*t}{(2D_x^*t+\sigma_x^2)}\right]}\right\}\exp\left[-\frac{1}{2}\frac{(\nu^*t-x)^2}{(2D_x^*t+\sigma_x^2)}\right].$$

Define $\Sigma^2 = 2[(\sigma_x^2D_x^*t)/(2D_x^*t + \sigma_x^2)]$ and $\mu = [\sigma_x^2(\nu^*t - x)]/(2D_x^*t + \sigma_x^2)$, then substitute into the exponential term above

$$\exp\left[-\frac{1}{2}\frac{(\xi+\mu)^2}{\Sigma^2}\right]\exp\left[-\frac{1}{2}\frac{(\nu^*t-x)^2}{(2D_x^*t+\sigma_x^2)}\right].$$

Substitute the above expression for the integrand of (A2) and rearrange to

$$X_5 = \exp\left[-\frac{1}{2}\frac{(\nu^*t-x)^2}{(2D_x^*t+\sigma_x^2)}\right]\frac{m_x}{\sqrt{2D_x^*t}}\frac{\Sigma}{\Sigma}\frac{1}{\sqrt{2\pi}}$$

$$\cdot\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}\frac{(\xi+\mu)^2}{\Sigma^2}\right]d\xi. \quad \text{(A3)}$$

Noting that $(1/\Sigma\sqrt{2\pi})\int_{-\infty}^{+\infty}\exp[-(1/2)(\xi+\mu)^2/\Sigma^2]\,d\xi \equiv 1$, (A3) simplifies to

$$X_5 = \frac{m_x\sigma_x}{\sqrt{2D_x^*t+\sigma_x^2}}\exp\left[-\frac{1}{2}\frac{(x-\nu^*t)^2}{(2D_x^*t+\sigma_x^2)}\right],$$

which was previously given as (12) in the text.

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